Effect of Eccentricity Fluctuations on Elliptic Flow

Color by Roberta Weir

Exploring the secrets of the universe

Art Poskanzer

The Berkeley School 2010

GSI-LBL Collaboration

July 1974 -- 36 years ago

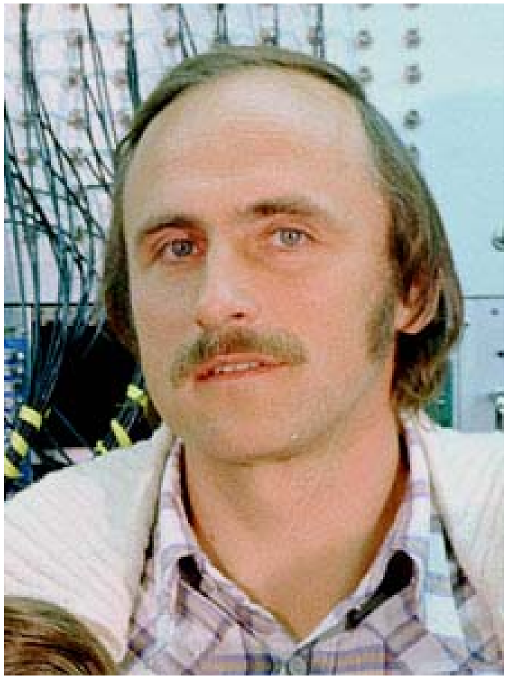
Reinhard and Rudolf Bock walked into my office

Plastic Ball Streamer Chamber

Hermann Grunder

2 Aug 74

Reinhard

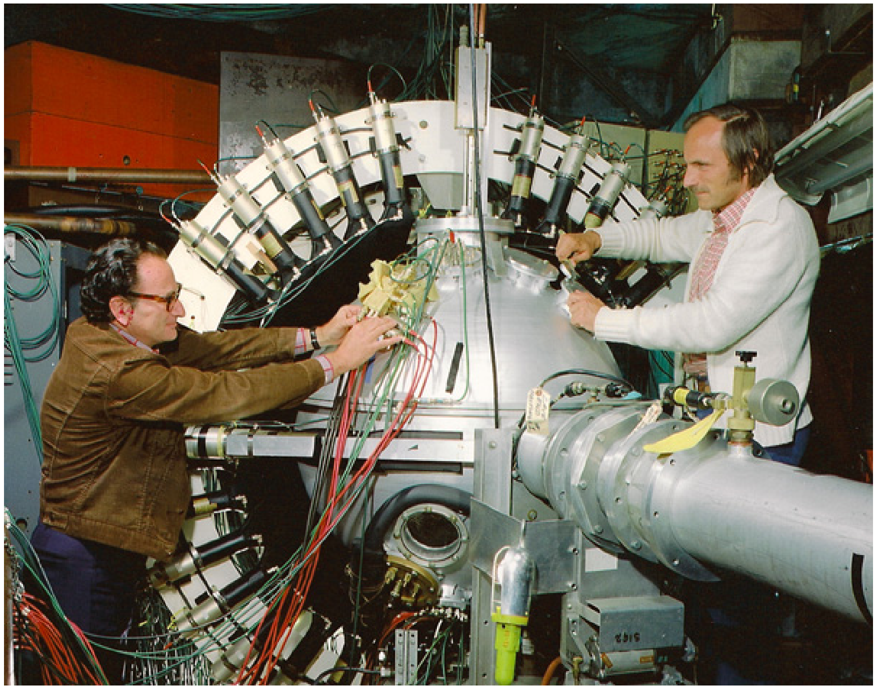


1976

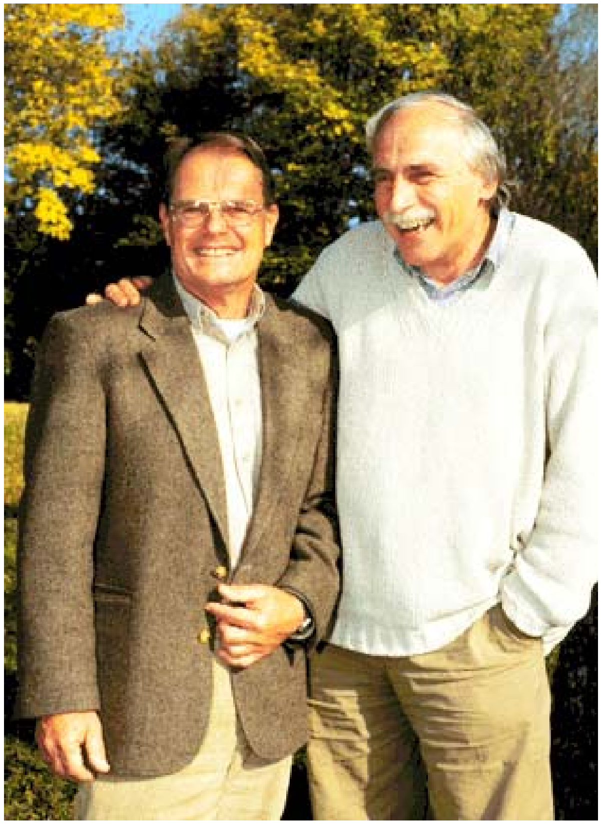
1995

2001

Rudolf Bock
1987



1977



1996

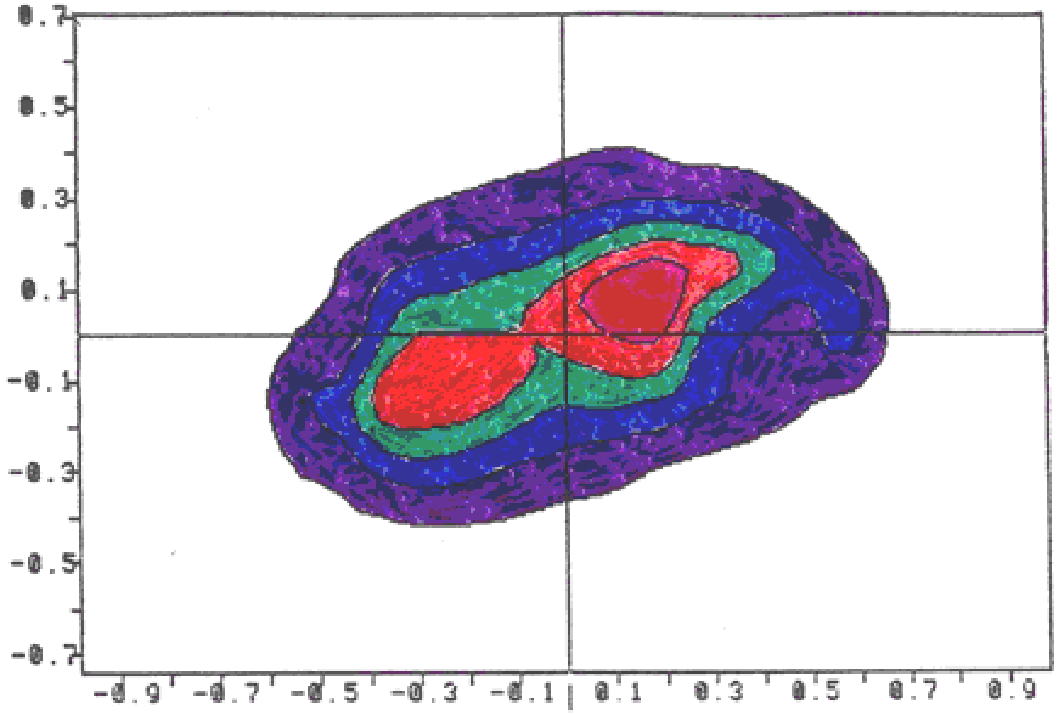
2001

photo by Jef Poskanzer

Filter theory to compare with data

p_x/A

transverse p
in event plane

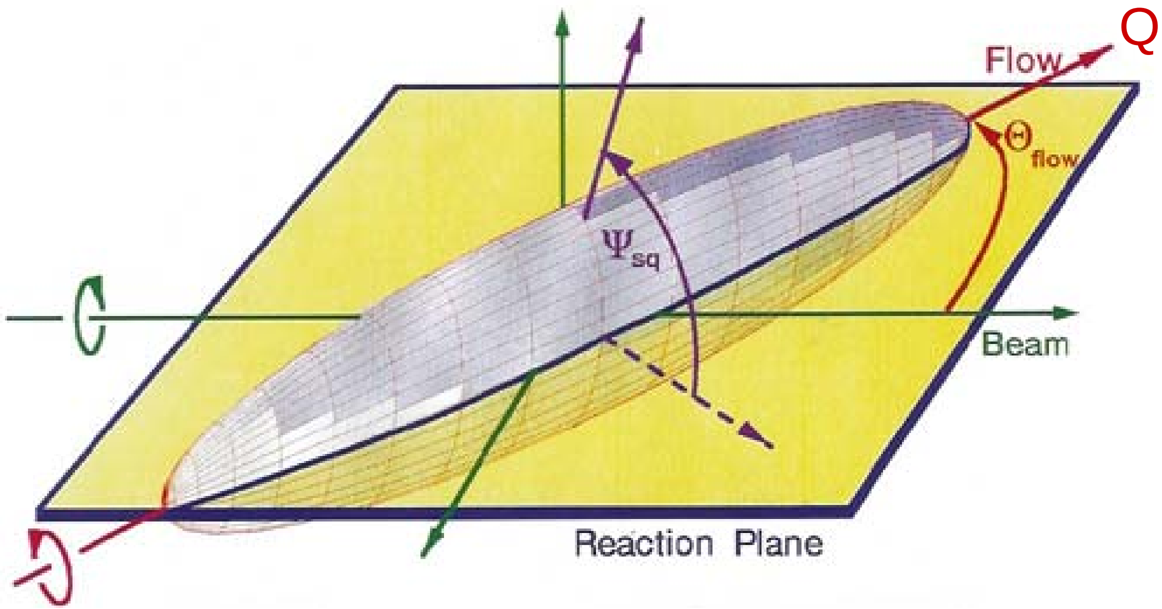


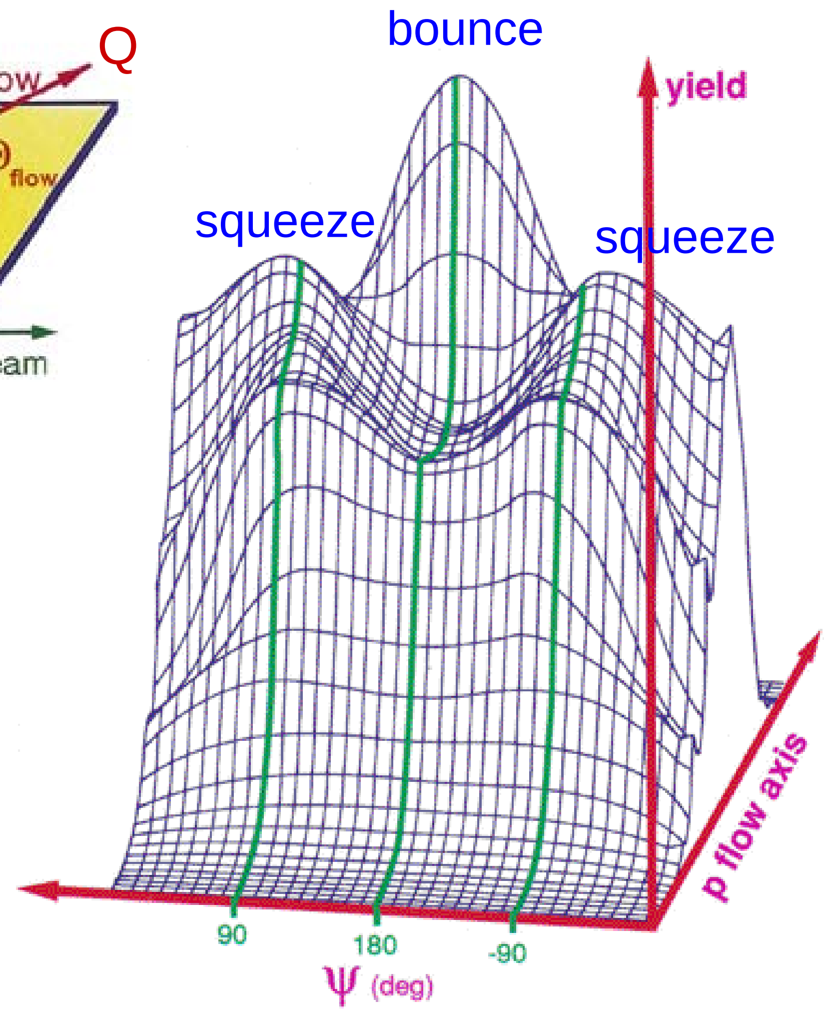
rapidity y/y_{proj}

Directed Flow

Plastic Ball, K.G.R. Doss et al., PRL **57**, 302 (1986)

Best Ellipsoid





Squeeze-out

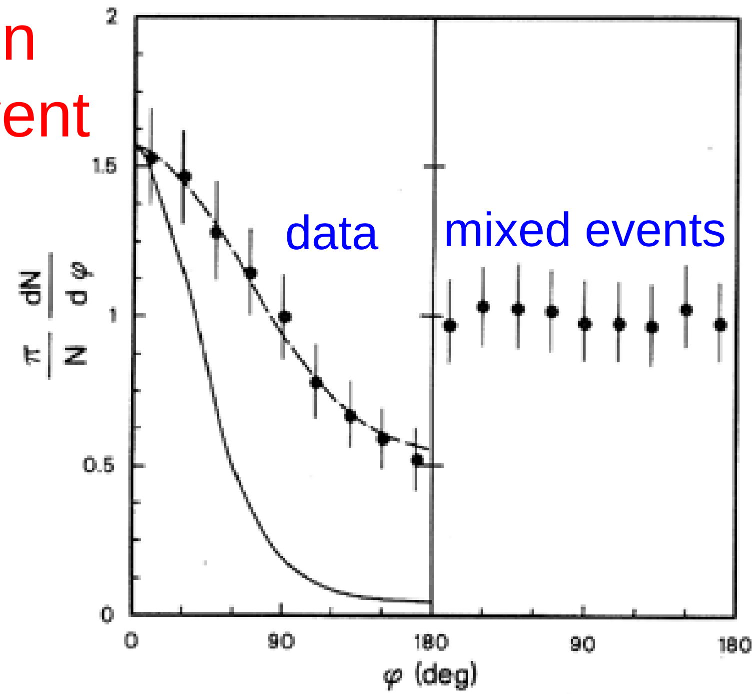
Negative elliptic flow

Plastic Ball, H.H. Gutbrod et al., PRC **42**, 640 (1991)
Diogene, M. Demoulins et al., Phys. Lett. **B241**, 476 (1990)
Plastic Ball, H.H. Gutbrod et al., Phys. Lett. **B216**, 267 (1989)

Analyze in the Transverse Plane

Transverse Momentum Analysis

correlation of sub-event planes



P. Danielewicz and G. Odyniec, Phys. Lett. **157B**, 146 (1985)

Fourier Harmonics

Fourier harmonics:

$$1 + 2v_1 \cos(\phi - \Psi_{RP}) + 2v_2 \cos[2(\phi - \Psi_{RP})] + \cdots$$

$$v_n = \langle \cos[n(\phi_i - \Psi_{RP})] \rangle$$

To remove acceptance correlations

Flatten event plane azimuthal distributions in lab

To measure event plane resolution

Correlate two independent sub-groups of particles

Event plane resolution correction made for each harmonic

Unfiltered theory can be compared to experiment!

Tremendous stimulus to theoreticians!

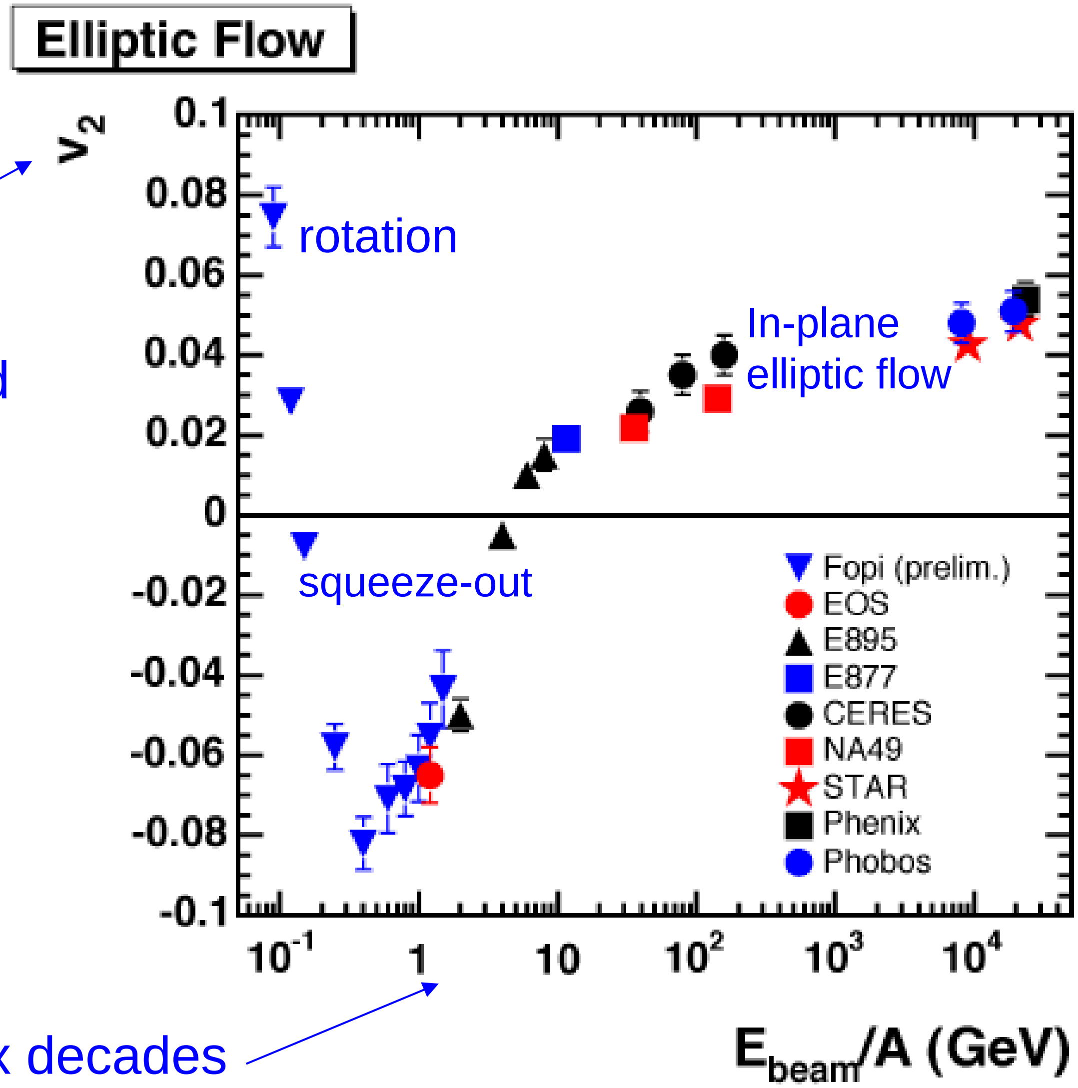
S. Voloshin and Y. Zhang, hep-ph/940782; Z. Phys. C **70**, 665 (1996)
See also, J.-Y. Ollitrault, arXiv nucl-ex/9711003 (1997)
and J.-Y. Ollitrault, Nucl. Phys. **A590**, 561c (1995)
A.M. Poskanzer and S.A. Voloshin, PRC **58**, 1671 (1998)

Elliptic Flow vs. Beam Energy

25% most central
mid-rapidity

all v_2

standard method



six decades

A. Wetzler (2005)

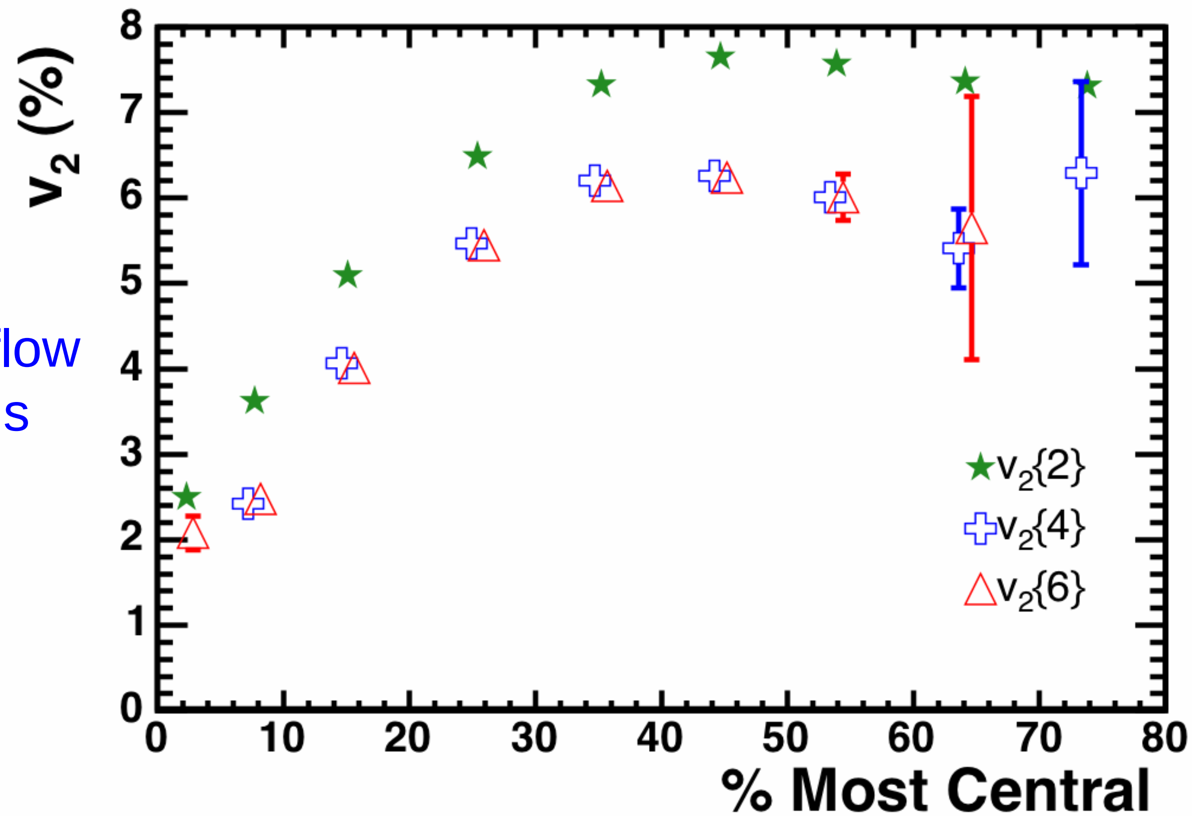
Generating Function Methods

$$\langle\langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle\rangle \equiv \langle u_{n,1}u_{n,2}u^*_{n,3}u^*_{n,4}\rangle - 2\langle u_{n,1}u^*_{n,2}\rangle^2 = -v_n^4\{4\}$$

Minimize complex generating function to evaluate four-particle correlation

Cumulants
Lee-Yang Zeros

Multi-particle methods eliminate 2-particle non-flow and Gaussian fluctuations



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Direct Four-Particle Correlation

$$\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle = \frac{(\sum e^{i2(\phi_i-\phi_j)})(\sum e^{i2(\phi_k-\phi_l)}) - degeneracies}{N(N-1)(N-2)(N-3)}$$

$$\begin{aligned} degeneracies = & \qquad \sum \left(e^{i2(\phi_i-\phi_j)}\right)^2 \\ + & \qquad 2\left(\sum e^{i2(\phi_i-\phi_j)} e^{i2(\phi_i-\phi_k)}\right) \\ + & \qquad 2\left(\sum e^{i2(\phi_i-\phi_j)} e^{i2(\phi_k-\phi_i)}\right) \end{aligned}$$

4-particle correlations without a generating function

Sergei Voloshin (2006)
Dhevan Gangadharan, thesis, UCLA (2010)

$v_2\{4\}$ Direct Cumulant

$$
\begin{aligned}
\left\langle\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = & \left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle \\
- & 2\left\langle e^{i2(\phi_1-\phi_3)}\right\rangle\left\langle e^{i2(\phi_2-\phi_4)}\right\rangle \\
- & \left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2(\phi_2-\phi_3-\phi_4)}\right\rangle \\
- & \left\langle e^{i2\phi_2}\right\rangle\left\langle e^{i2(\phi_1-\phi_3-\phi_4)}\right\rangle \\
- & 2\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_1+\phi_2-\phi_4)}\right\rangle \\
- & \left\langle e^{i2(\phi_1+\phi_2)}\right\rangle\left\langle e^{-i2(\phi_3+\phi_4)}\right\rangle \\
+ & 4\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_2-\phi_4)}\right\rangle \\
+ & 4\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{i2(\phi_1-\phi_4)}\right\rangle \\
+ & 2\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2(\phi_3+\phi_4)}\right\rangle \\
+ & 2\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{-i2\phi_4}\right\rangle\left\langle e^{i2(\phi_1+\phi_2)}\right\rangle \\
- & 6\left\langle e^{i2\phi_1}\right\rangle\left\langle e^{i2\phi_2}\right\rangle\left\langle e^{-i2\phi_3}\right\rangle\left\langle e^{-i2\phi_4}\right\rangle
\end{aligned}
$$

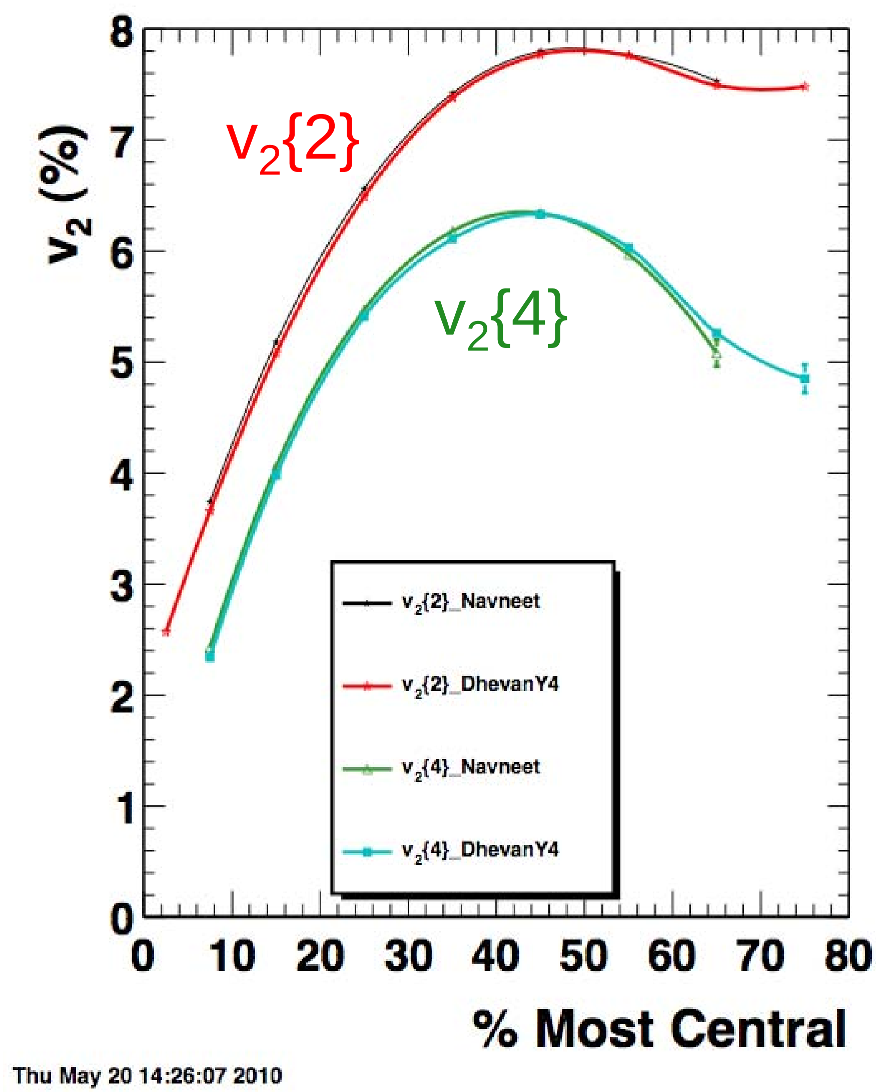
acceptance corrections

$$\left\langle\left\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = -v_2^4\{4\}$$

Dhevan Gangadharan, thesis, UCLA (2010)

Direct $v_2\{4\}$

Two independent implementations agree



STAR preliminary

Sergei Voloshin, STAR
Dhevan Gangadharan, STAR
Navnett Pruthi, STAR
Ante Bilandzic, ALICE

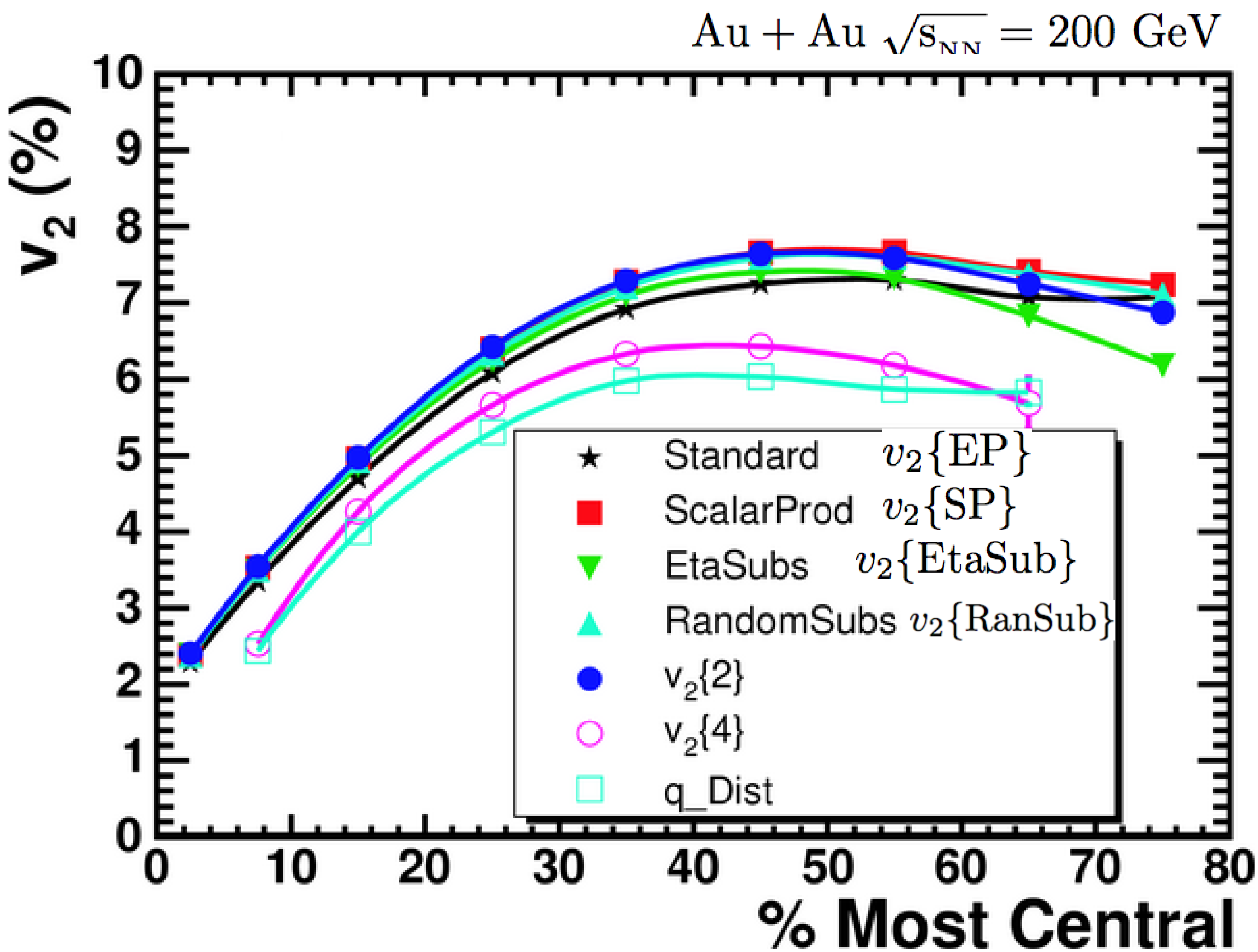
Methods

- "Two-particle":
  - $v_2\{2\}$: each particle with every other particle
  - $v_2\{subEP\}$: each particle with the EP of the other subevent
  - $v_2\{EP\}$ "standard": each particle with the EP of all the others
  - $v_2\{SP\}$: same, weighted with the length of the Q vector

- Many-particle:
  - $v_2\{4\}$: 4-particle - 2 * (2-particle)^2
    - Generating function or Direct Cumulant
  - $v_2\{q\}$: distribution of the length of the Q vector
  - $v_2\{LYZ\}$: Lee-Yang Zeros multi-particle correlation

review of azimuthal anisotropy:
arXiv: 0809.2949

Integrated v_2



STAR, J. Adams et al., PRC **72**, 014904 (2005)

Measurements

- **Two-particle methods**
  - **contain nonflow** $\left\langle \cos\phi_1 - \cos\phi_2 \right\rangle = \left\langle v^2 \right\rangle + \delta \longleftarrow$ nonflow
  - **mean of some power of the distribution in the Participant Plane** $v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$

- **Multi-particle methods**
  - **suppress nonflow**
  - **mean in the Reaction Plane in Gaussian approx.**

Bhalerao and Ollitrault, Phys Lett. **B 641**, 260 (2006)
Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

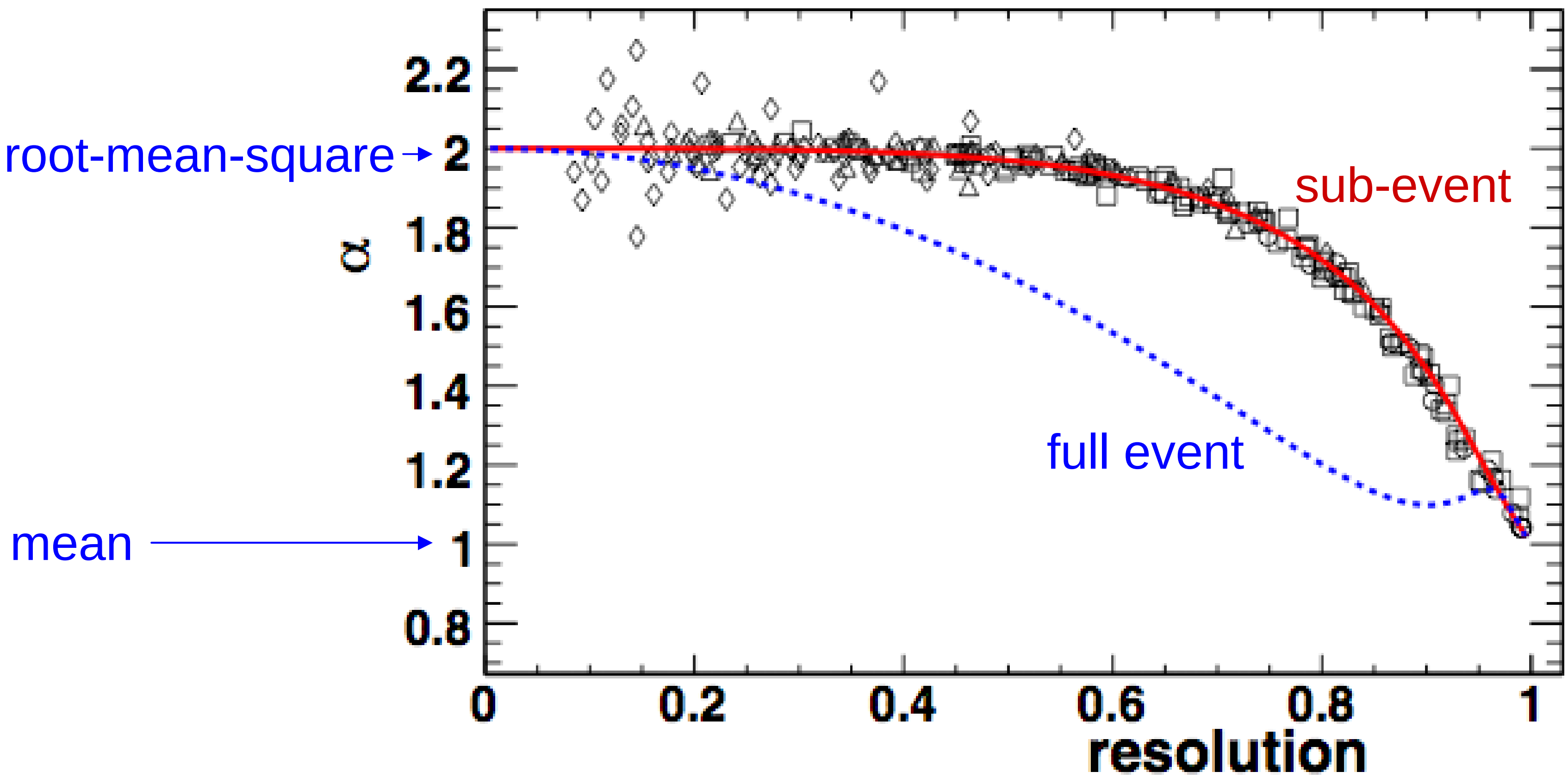
Effect of Eccentricity Fluctuations on Elliptic Flow

- Mean of some power of the distribution

- Participant plane fluctuations

Effect of Fluctuations on the Mean

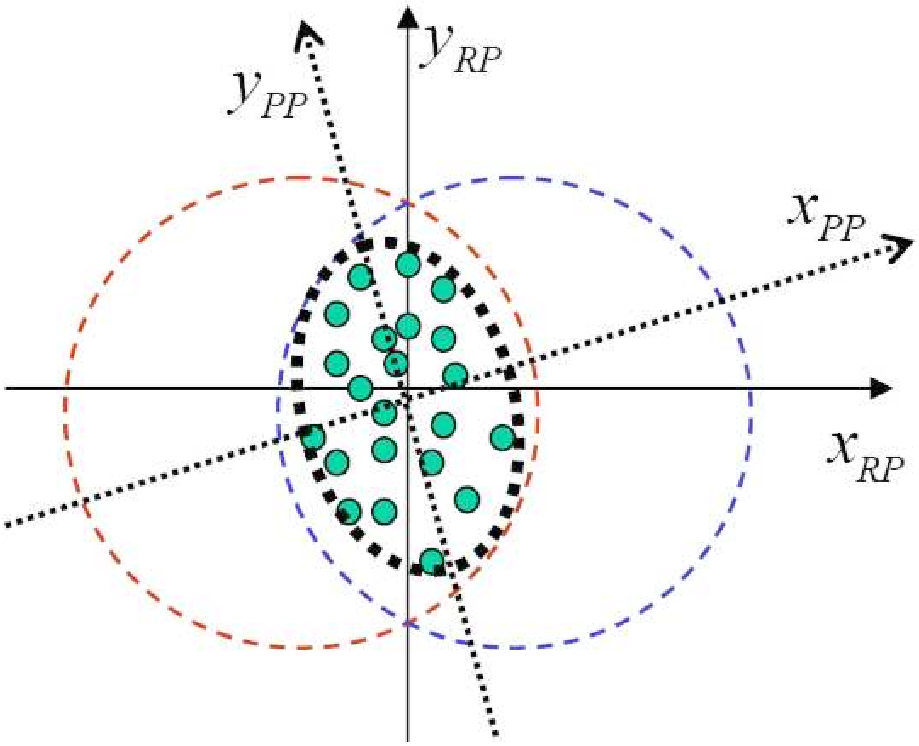
$$v_2\{\ \} = \langle v^\alpha \rangle^{1/\alpha}$$



$$v_2\{\ \}^2 = \langle v \rangle^2 + (\alpha - 1)\sigma_v^2$$

Points: simulations by PHOBOS+
Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

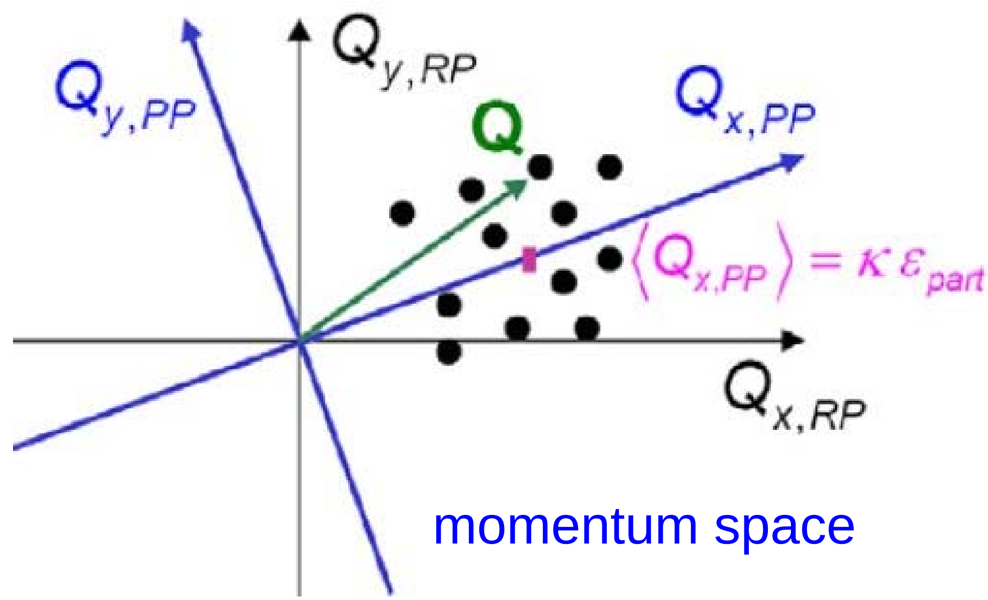
Reaction, Participant, and Event Planes



determined by particles struck

determined by impact parameter

coordinate space



participant plane

momentum space

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

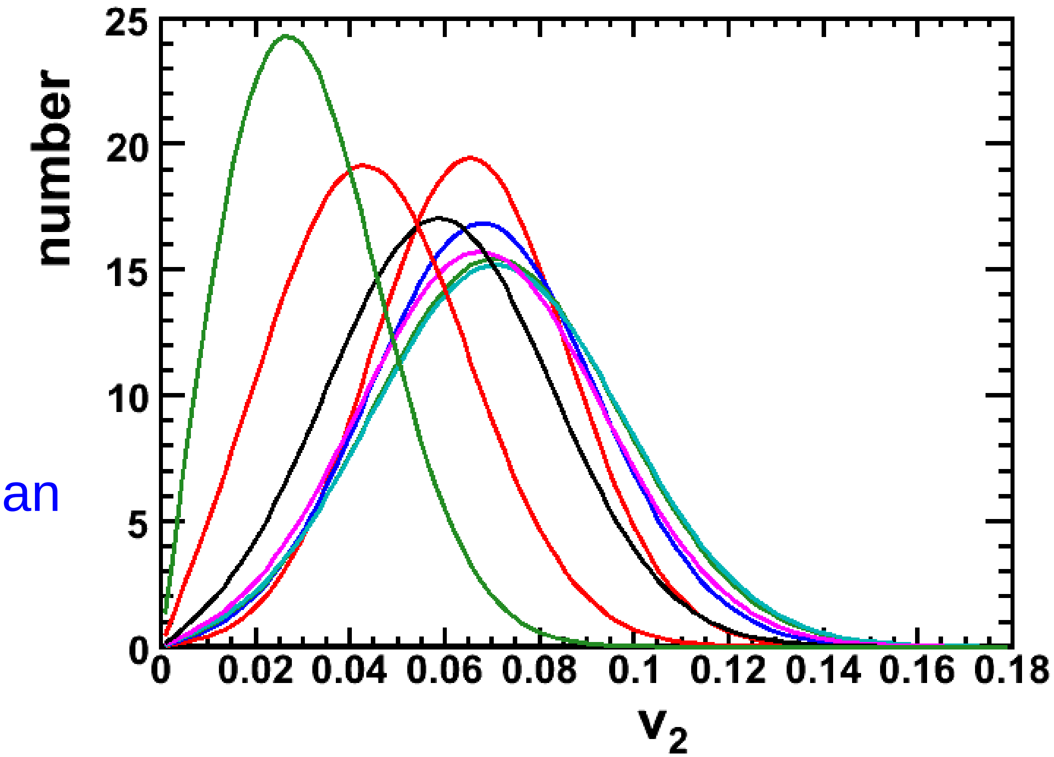
v_2 Fluctuations from ε_{part} Fluctuations

Assume width with same percent width as ε_{part}: $$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$$

σ_ε is from standard deviation of nucleon MC Glauber of ε_{part}

Bessel-Gaussian: $$\frac{dn}{dv} = \frac{v}{\sigma_0^2} I_0 \left(\frac{v\ v_0}{\sigma_0^2} \right) \exp \left(-\frac{v^2 + v_0^2}{2\sigma_0^2} \right)$$

2D Gaussian fluctuations in ε_x and ε_y in the reaction plane lead to Bessel-Gaussian fluctuations along the participant plane axis



Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

Theory is in RP, except...

- **Event-by-Event without impact parameter**

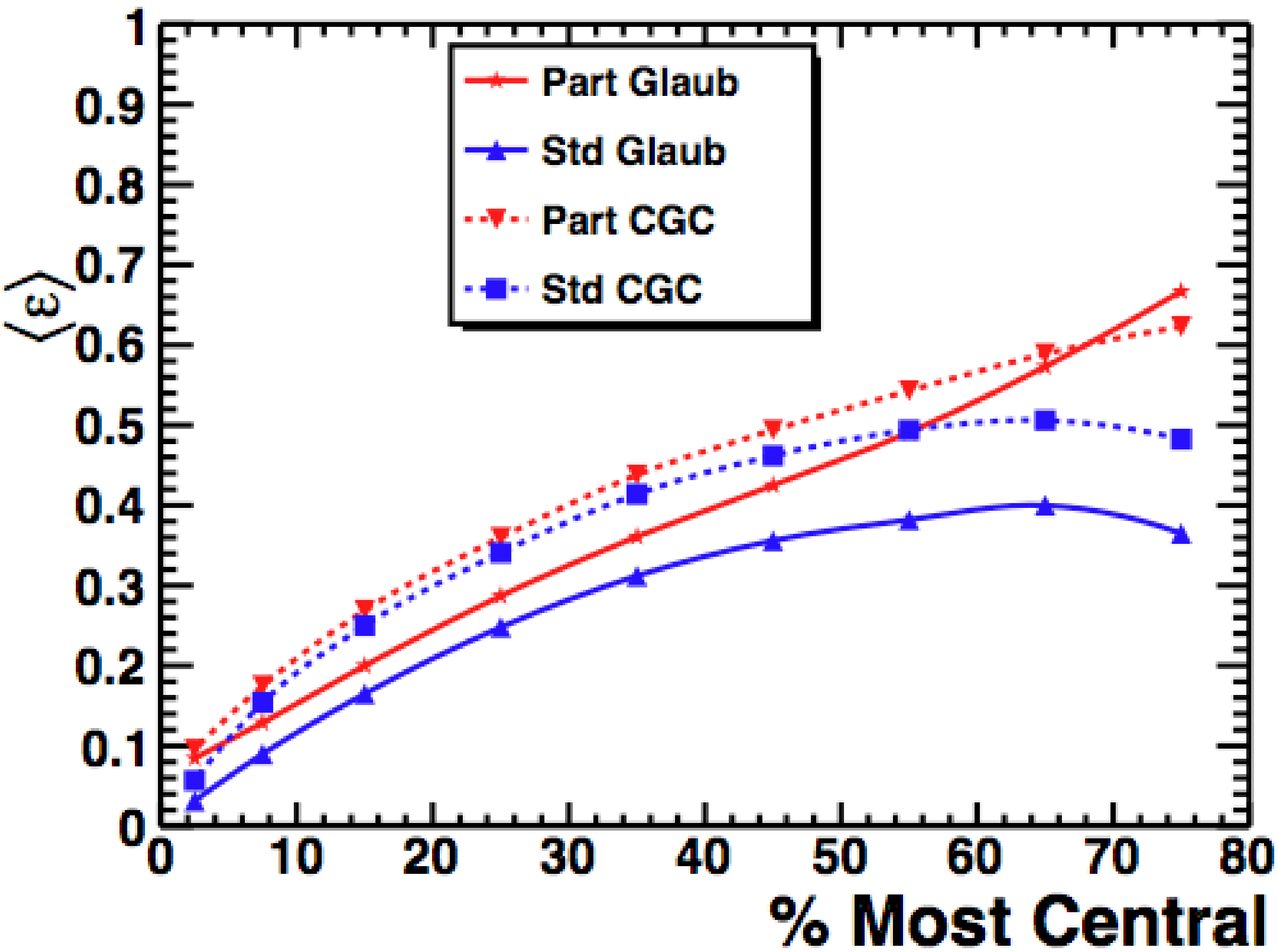
- **Kodama**
  - **NeXSPheRIO**
  - **Hydro for event-by-event participants**

- **Hirano**
  - **Determine PP for each event**
  - **Rotate event to RP**
  - **Thus include PP fluctuations in initial conditions**

R. Andrade et al., Phys. Rev. Lett. **97**, 202302 (2006)

T. Hirano and Y. Nara, PRC **79**, 064904 (2009)

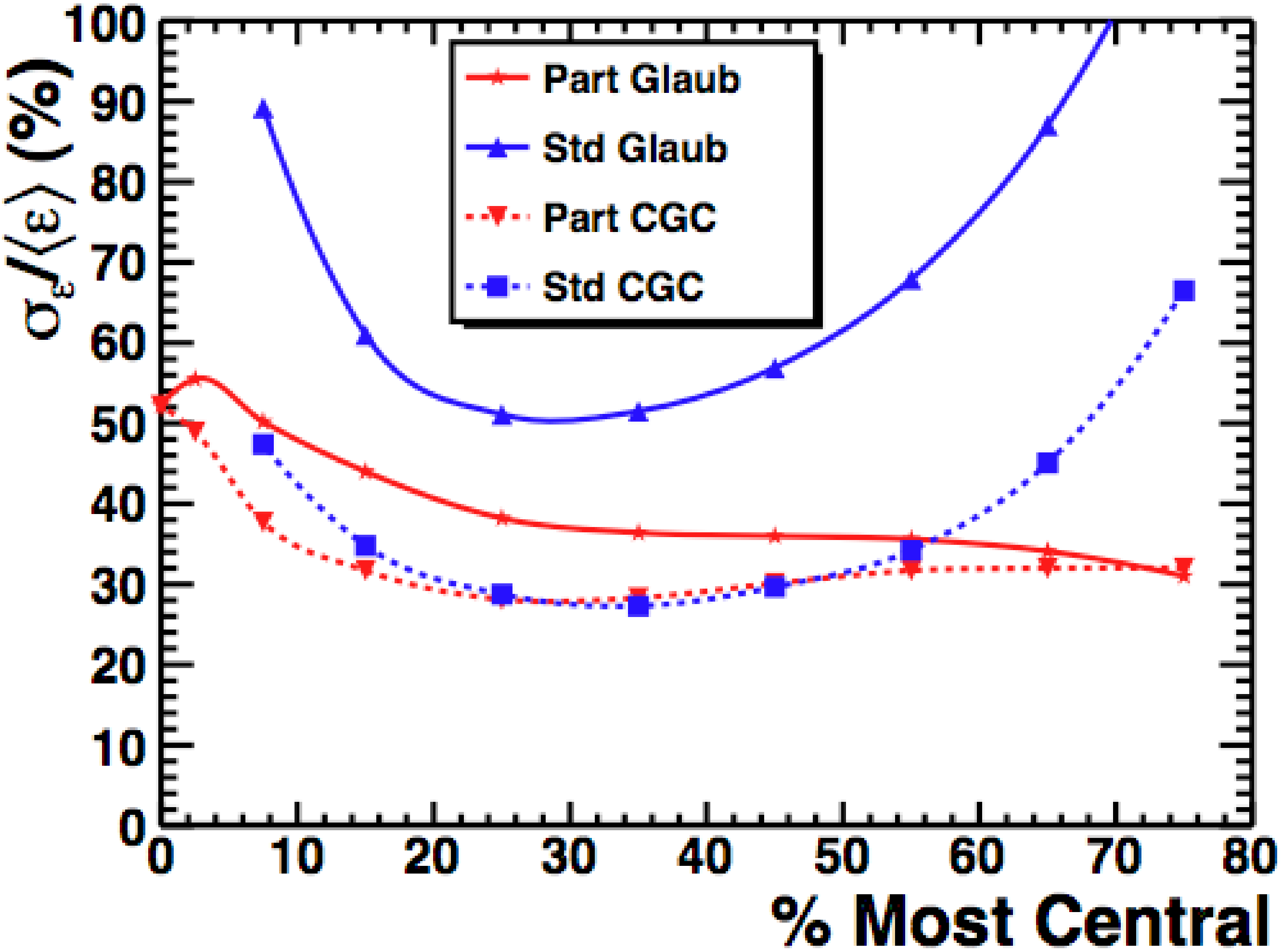
Eccentricities



Hiroshi Masui

Drescher and Nara, PRC **76**, 041903(R) (2007)

Eccentricity Fluctuations



Hiroshi Masui
Drescher and Nara, PRC **76**, 041903(R) (2007)

An Application to Data

- **Correct for nonflow**

$$\langle \cos\phi_1 - \cos\phi_2 \rangle = \langle v^2 \rangle + \delta$$ nonflow

- **Correct to mean v_2 in PP**
- **Correct to RP**
- **Assumptions**

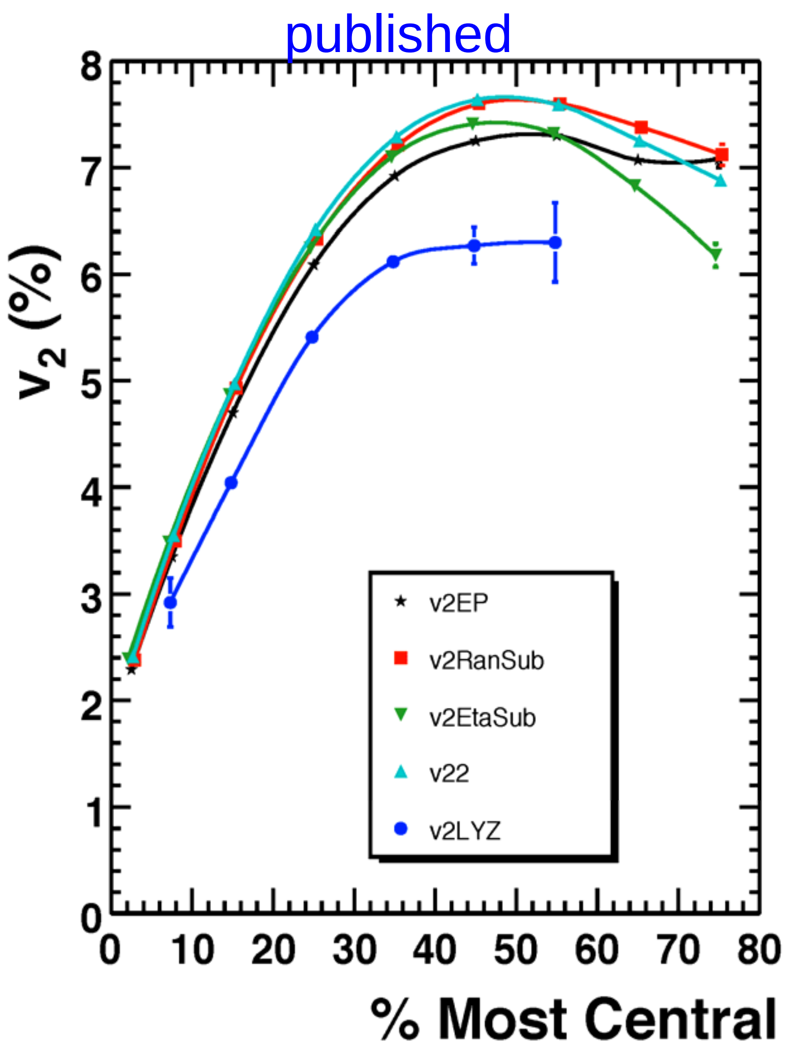
$$\sigma_{v2} = \frac{\sigma_\varepsilon}{\langle \varepsilon \rangle} \langle v_2 \rangle$$ MC ε participant

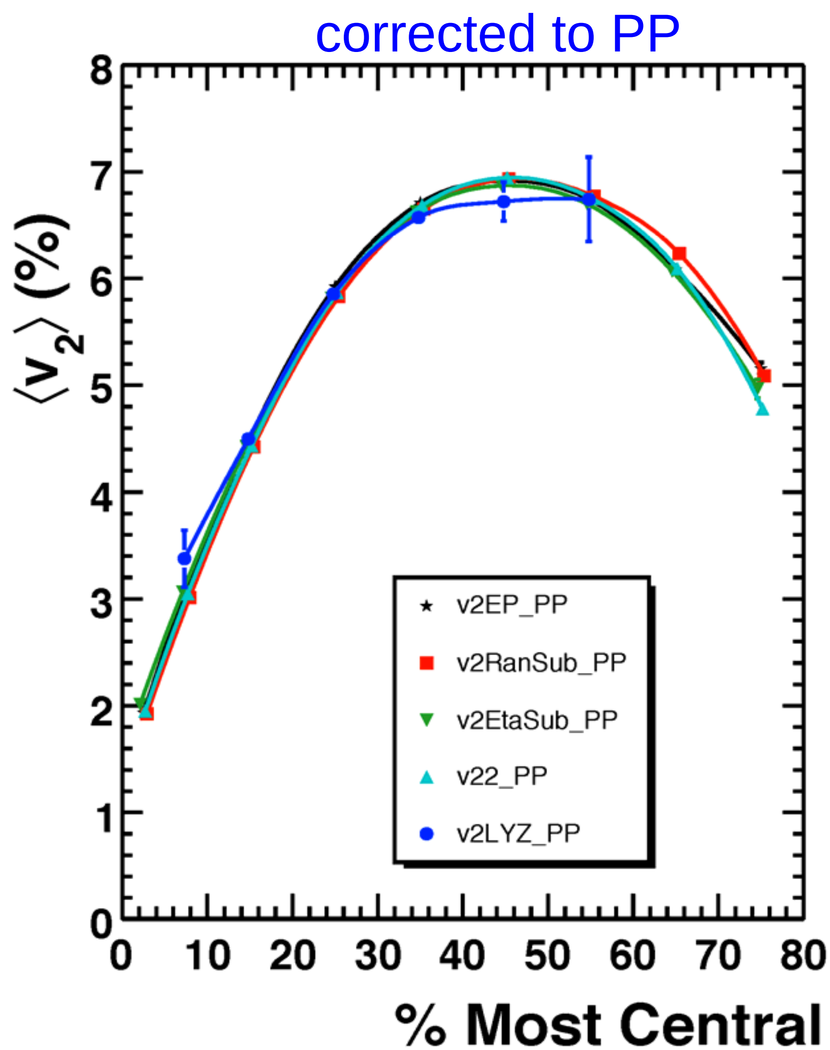
$$\delta_2 = 2\ \delta_{pp}/N_{\mathrm{part}} \qquad \delta_{pp} = 0.0145$$

$$\delta_{\mathrm{etaSub}} = 0.5\ \delta_2$$ less nonflow

Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

Data Corrected to $<v_2>$





agreement for mean v_2
in participant plane

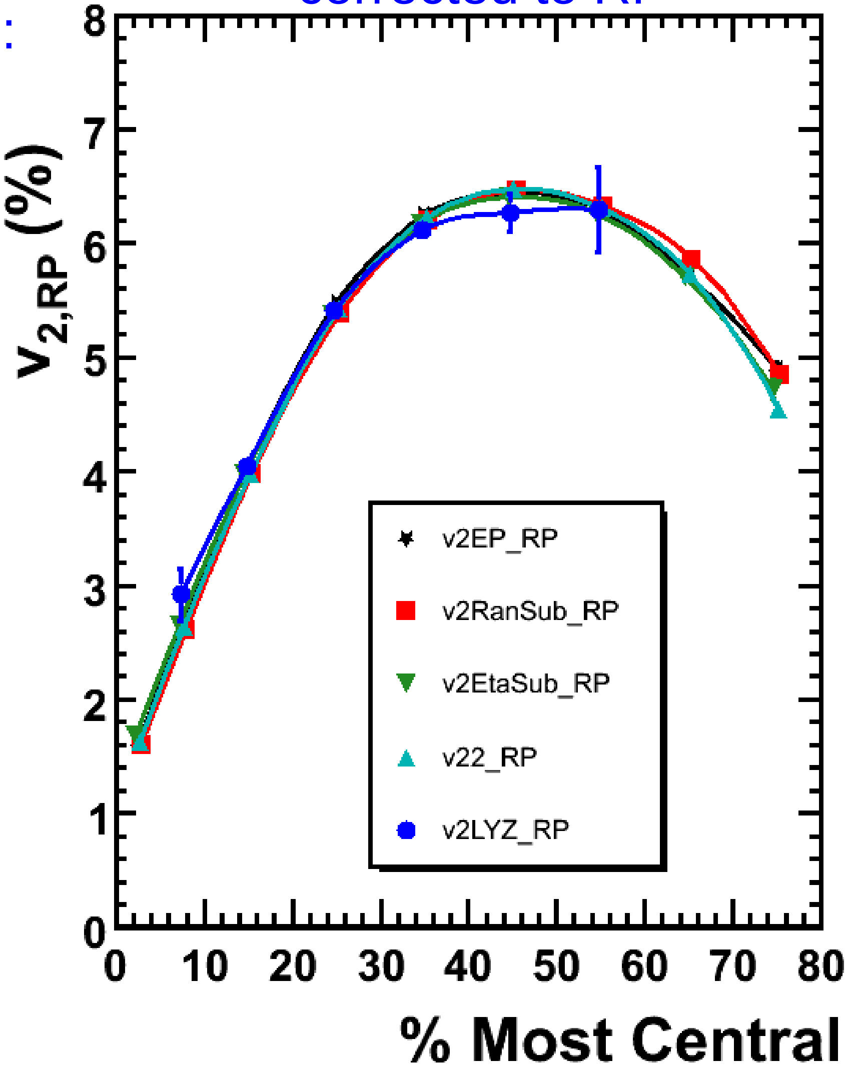
Ollitrault, Poskanzer, and Voloshin, PRC **80**, 014904 (2009)

v_2 in the Reaction Plane

in Gaussian fluctuation approximation:

$$\langle v_{2,\mathrm{PP}} \rangle^2 \simeq \langle v_{2,\mathrm{RP}} \rangle^2 + \sigma^2_{v2,\mathrm{part}}$$

corrected to RP



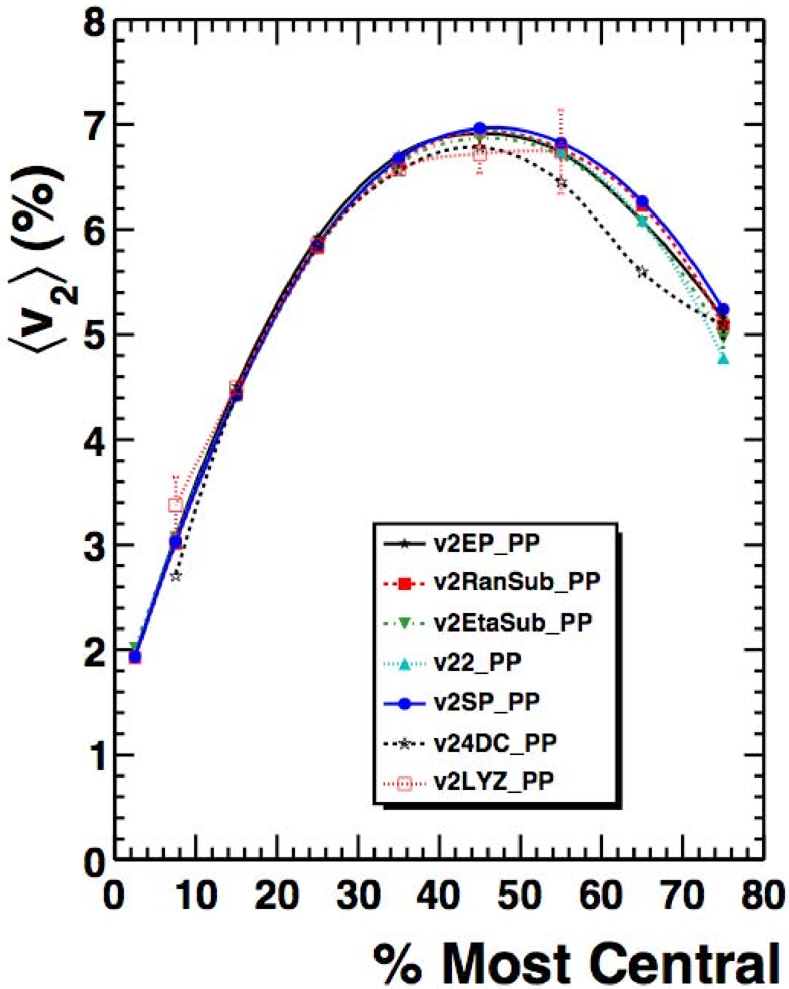
Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

New Results

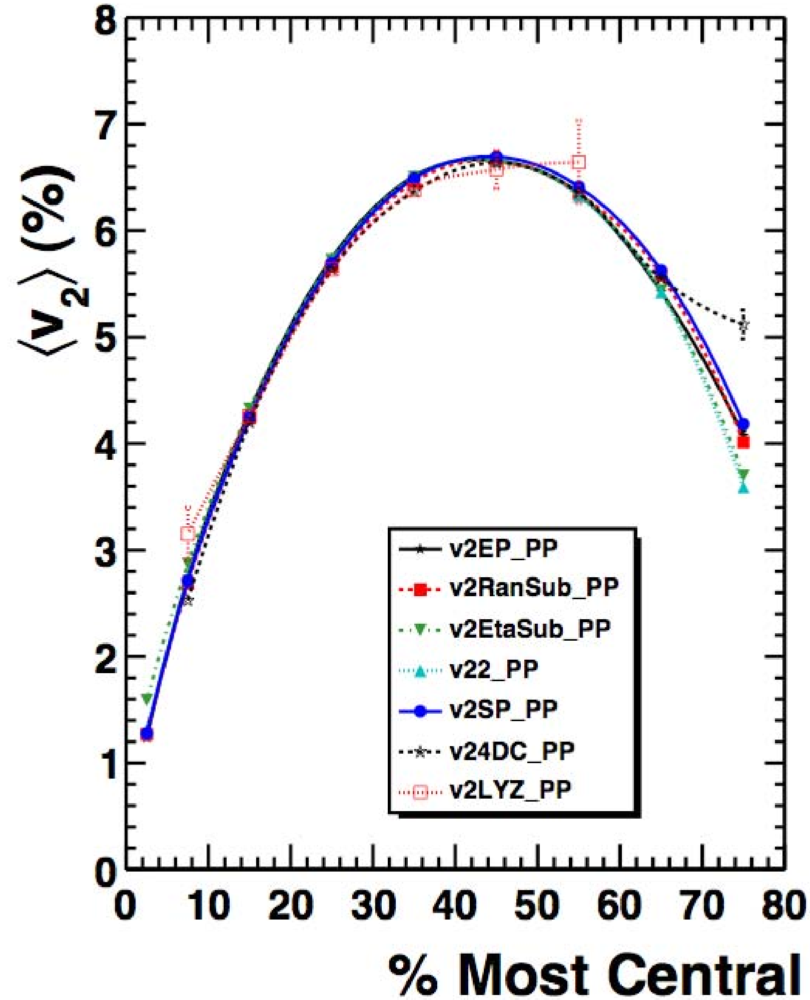
- **Direct Cumulants**
- **Non-Gaussian behavior**

Participant Plane

Glauber



CGC



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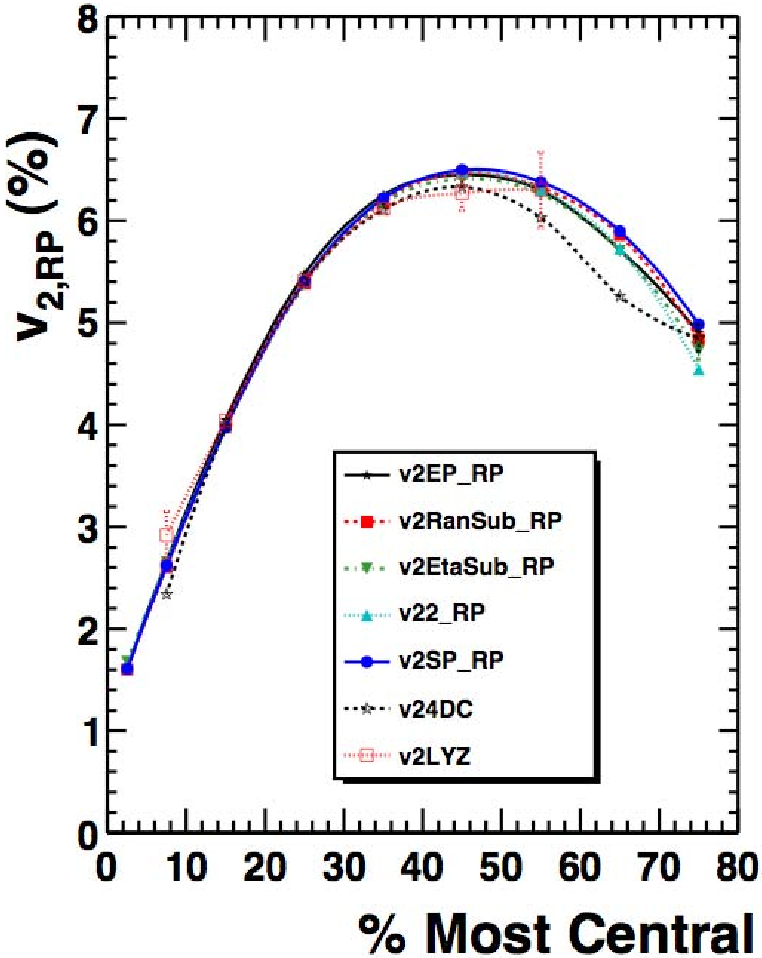
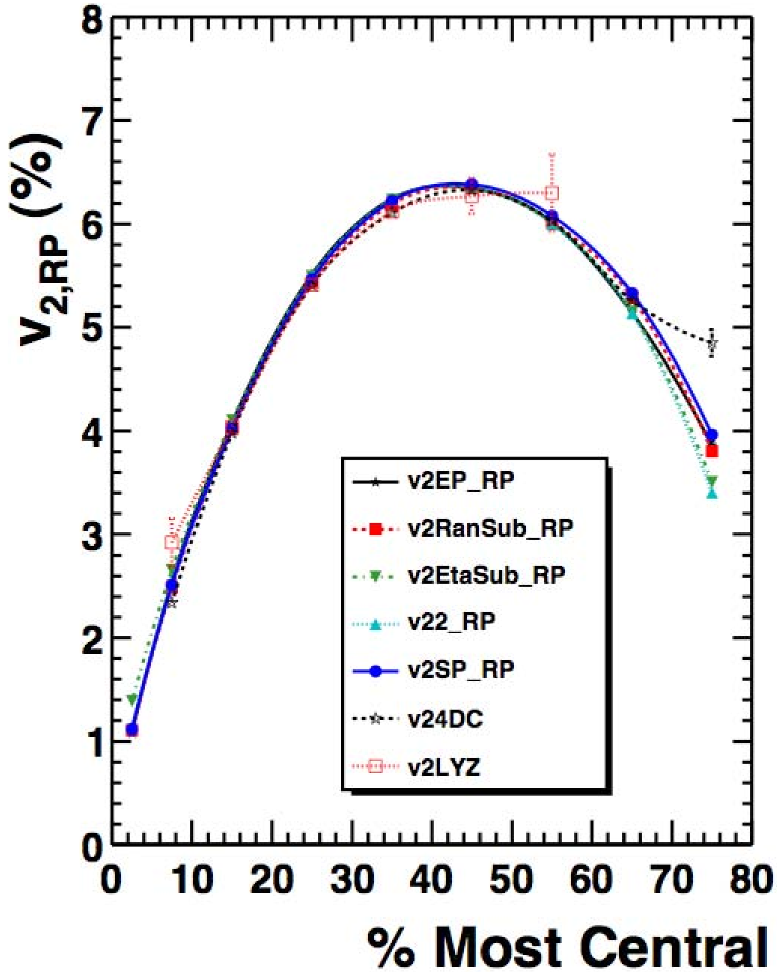
Reaction Plane

$$v_{RP} = v_{PP}\sqrt{1-(\sigma_\varepsilon/\varepsilon)^2}$$

Glauber



CGC



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Can Compare to Theory

Because we now:

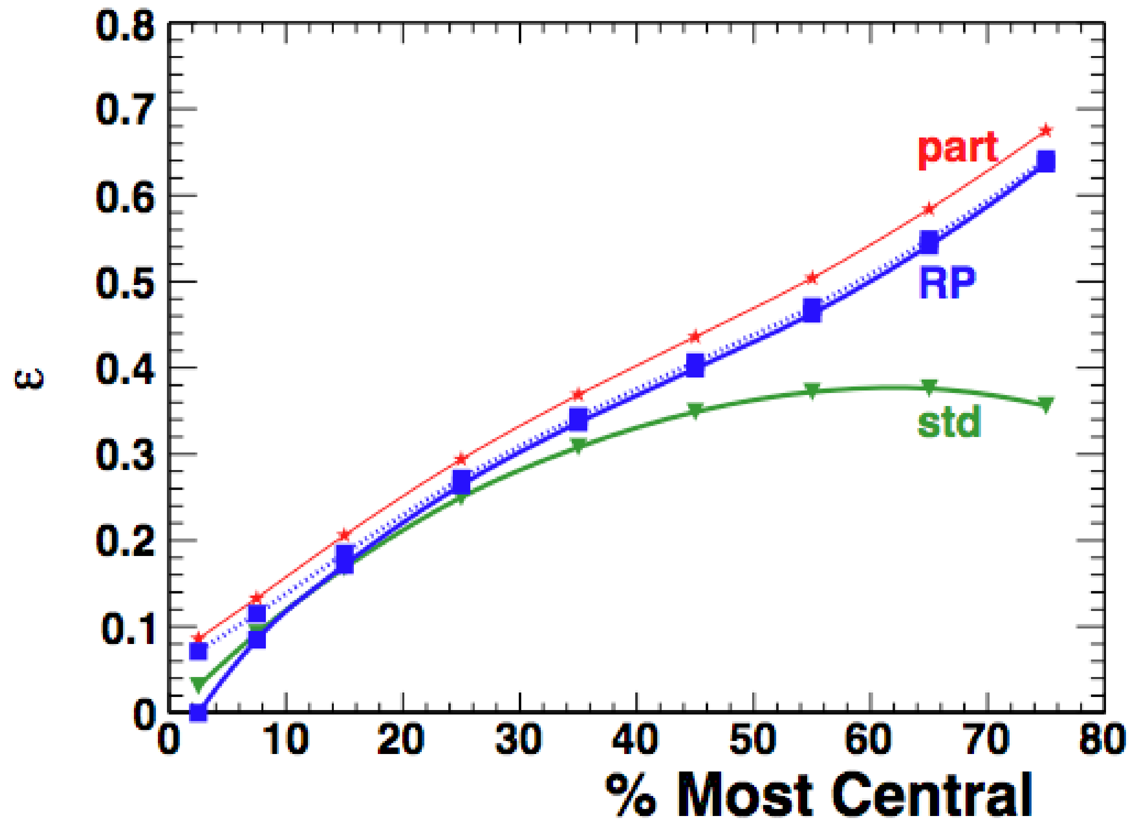
- **Remove acceptance correlations**
- **Correct for Event Plane resolution**
- **Correct for mean of a power of the distribution**
- **Correct for fluctuations of the PP**

Test Method Using ε_{part} to ε_{std}

dashed blue uses $\varepsilon_{RP} = \varepsilon_{PP}\sqrt{1-(\sigma_{\varepsilon,PP}/\varepsilon_{PP})^2}$

solid blue uses exact equations in Gaussian Model paper

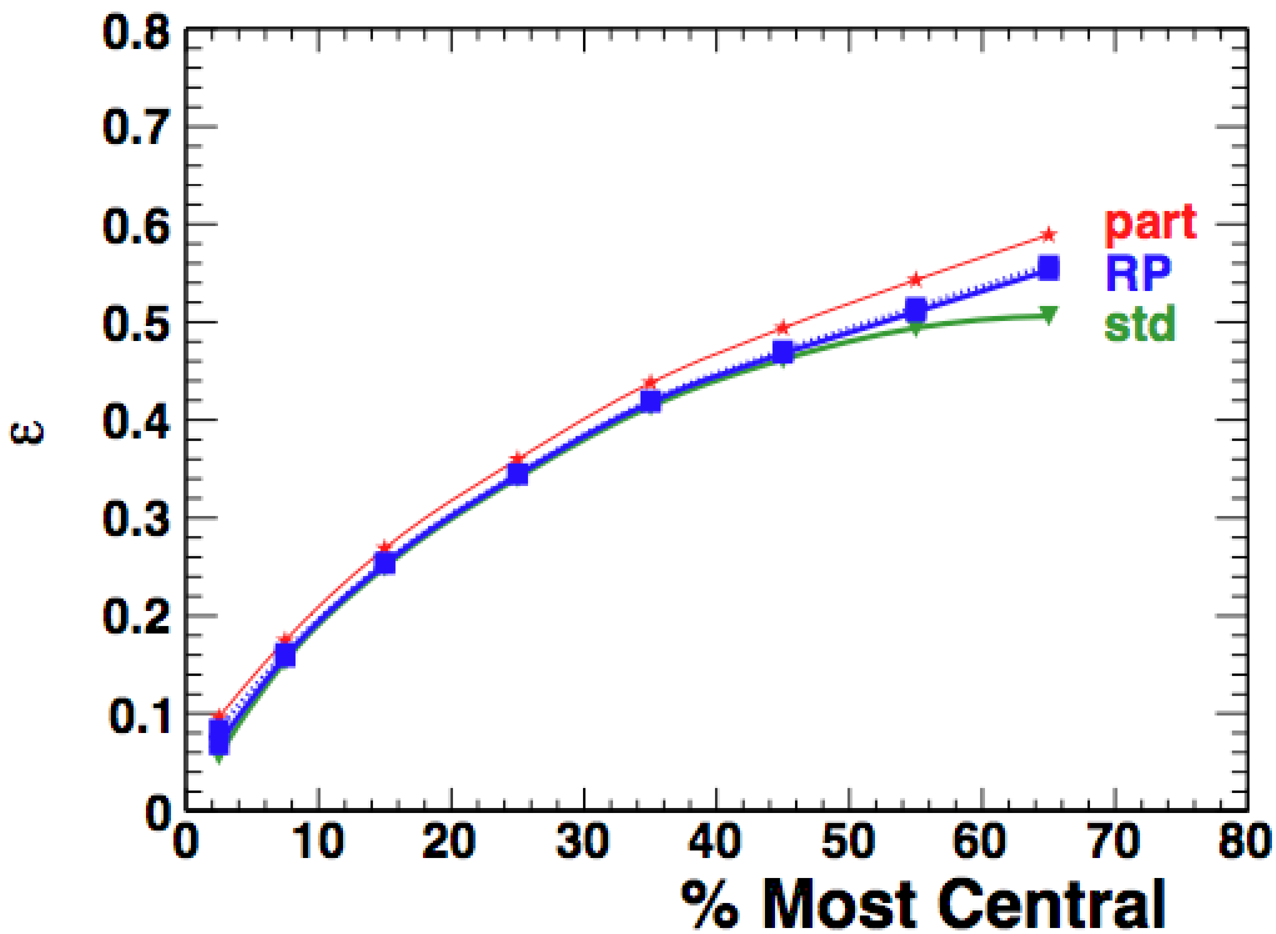
Glauber



Why does blue not go down to green?

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

CGC

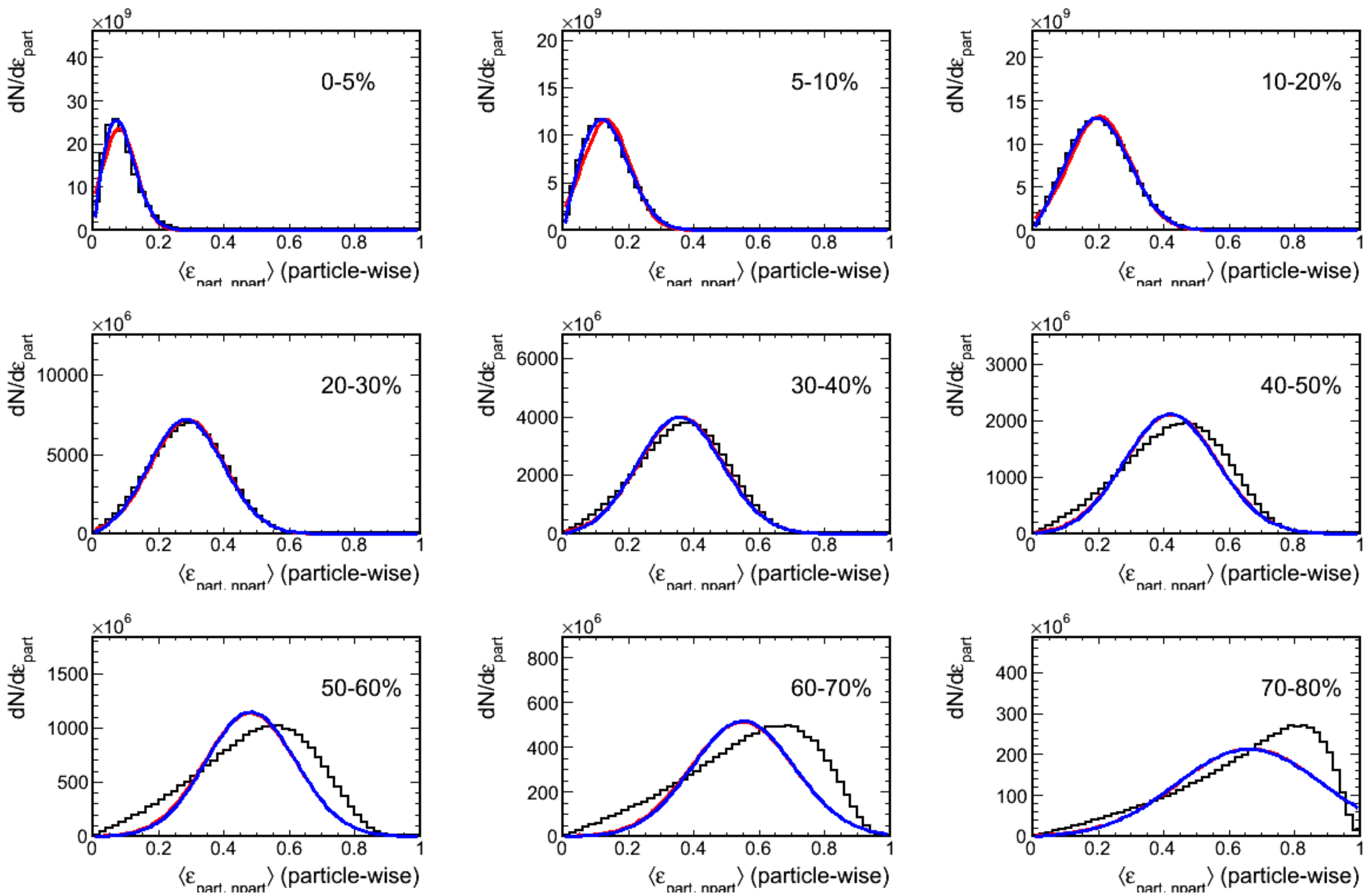


CGC participant fluctuations less
part and std much closer together
so can say RP = std to more peripheral collisions

Voloshin, Poskanzer, Tang, and Wang, Phys. Lett. B **659**, 537 (2008)

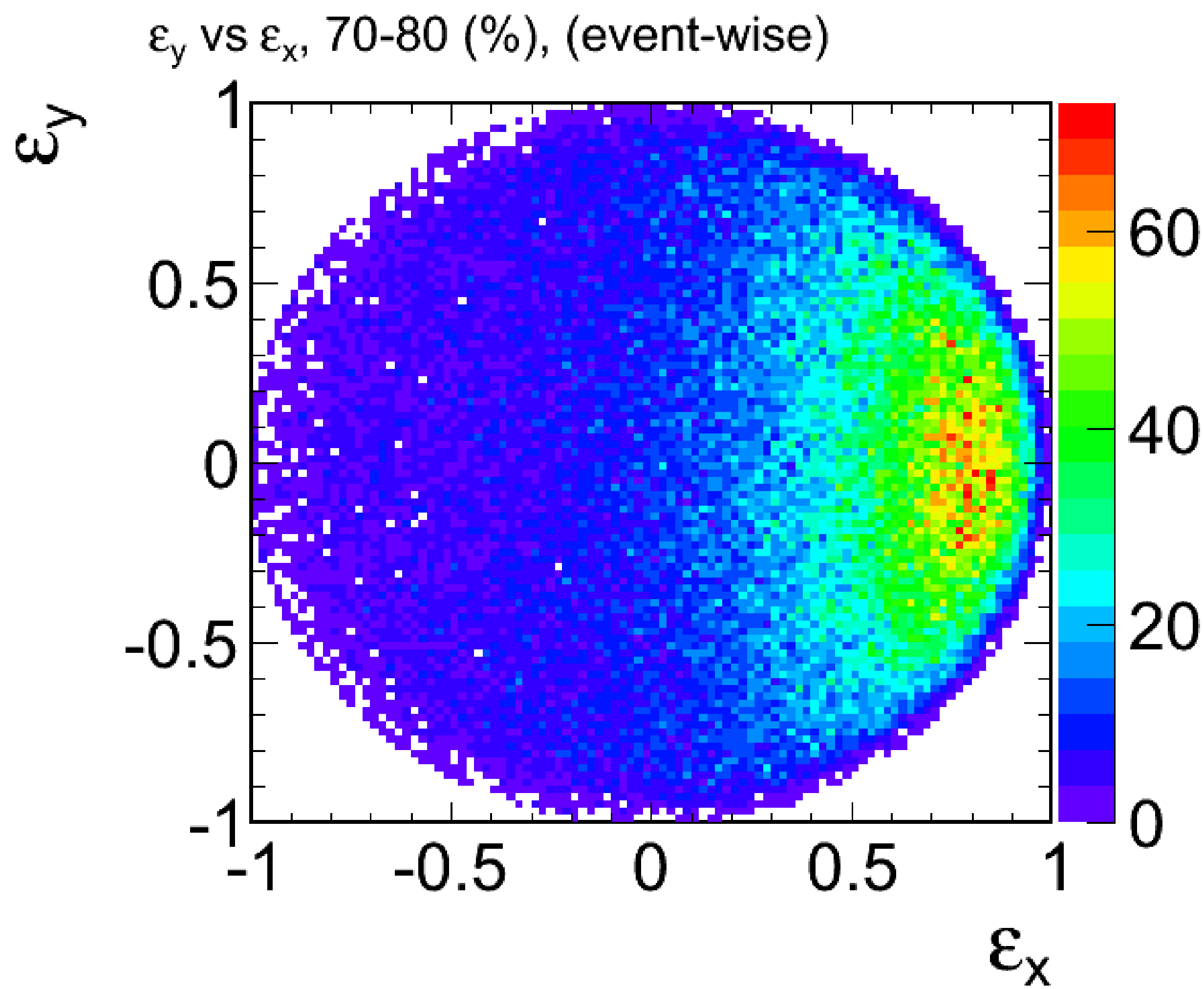
Glauber ε_{part} Distributions

Gaussian and Bessel-Gaussian fits to the black calculations



Non-Gaussian for peripheral collisions

$\varepsilon_{part} < 1$



$\boldsymbol{\varepsilon}$ can not be greater than 1

Status

- **Even though the eccentricity distribution is not Gaussian, still could be:**

$$v_2 \propto \varepsilon_2 \qquad\qquad \sigma_{v2} = \frac{\sigma_\varepsilon}{\langle\varepsilon\rangle}\langle v_2\rangle$$

- **However,**

$$v_2\{4\} \neq v_{2,\mathrm{RP}}$$

- **for peripheral collisions**
  - **as estimated from Glauber calculations**

Emphasize Direct $v_2\{4\}$

- No Event Plane
- Corrects for acceptance correlations
- One pass through the data
- Eliminates 2-particle nonflow correlations
- Gives mean of the distribution
- Gives v_2 in the RP
  - except for peripheral collisions